5-8 Rational Zero Theorem

List all of the possible rational zeros of each function.

1. $f(x) = x^3 - 6x^2 - 8x + 24$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 24 and q is a factor of 1.

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$
$q: \pm 1$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

2. $f(x) = 2x^4 + 3x^2 - x + 15$

SOLUTION:

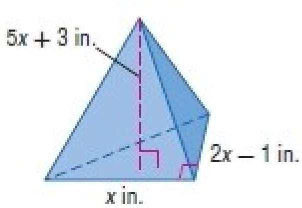
If $\frac{p}{q}$ is a rational zero, then p is a factor of 15 and q is a factor of 2.

$p: \pm 1, \pm 3, \pm 5, \pm 15$
$q: \pm 1, \pm 2$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$$

3. **CCSS REASONING** The volume of the triangular pyramid is 210 cubic inches. Find the dimensions of the solid.



SOLUTION:
Use the formula to find the volume of the triangular pyramid.
The volume of the pyramid is:

$$V(x) = \frac{1}{3}B \cdot h \quad \text{Formula for the volume of a triangular pyramid}$$
$$= \frac{1}{3}\left(\frac{1}{2}(2x-1)x\right)(5x+3) \quad \text{Subsitute.}$$
$$= \frac{1}{6}(2x^2 - x)(5x+3) \quad \text{Multiply.}$$
$$= \frac{1}{6}(10x^3 + x^2 - 3x) \quad \text{Multiply.}$$

Set the volume of pyramid as 210 cm^3.

$$\frac{1}{6}(10x^3 + x^2 - 3x) = 210 \quad \text{Volume is 210.}$$
$$10x^3 + x^2 - 3x = 1260 \quad \text{Multiply.}$$
$$10x^3 + x^2 - 3x - 1260 = 0 \quad \text{Subtract 1260 from each side.}$$

The possible rational zeros are:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 9, \pm 10, \pm 12,$$
$$\pm 14, \pm 15, \pm 18, \pm 20, \pm 21, \pm 30, \pm 35, \pm 36, \pm 42,$$
$$\pm 60, \pm 63, \pm 70, \pm 84, \pm 90, \pm 105, \pm 126, \pm 140,$$
$$\pm 180, \pm 210, \pm 252, \pm 315, \pm 420, \pm 630, \pm 1260,$$
$$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{7}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{21}{2}, \pm\frac{35}{2}, \pm\frac{63}{2},$$
$$\pm\frac{105}{2}, \pm\frac{315}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{7}{5}, \pm\frac{9}{5},$$
$$\pm\frac{12}{5}, \pm\frac{14}{5}, \pm\frac{18}{5}, \pm\frac{21}{5}, \pm\frac{36}{5}, \pm\frac{42}{5}, \pm\frac{63}{5}, \pm\frac{84}{5},$$
$$\pm\frac{126}{5}, \pm\frac{252}{5}, \pm\frac{1}{10}, \pm\frac{3}{10}, \pm\frac{7}{10}, \pm\frac{9}{10}, \pm\frac{21}{10}, \pm\frac{63}{10}$$

There is one change of sign of the coefficients, so by Descartes' Rule of sign, there is only one positive zero.
Use synthetic division to find the zero.

f	10	1	−3	−1260
5	10	51	252	0

The only one positive zero of the function is 5. We don't need to check other zeros as the length cannot be negative.
Therefore, the dimensions are 5 in.× 9 in. × 28 in.

Find all of the rational zeros of each function.

4. $f\ (x) = x^3 - 6x^2 - 13x + 42$

SOLUTION:

$p: \pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$
$q: \pm 1$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$$

Make a table for synthetic division and test possible zeros.

$\frac{p}{q}$	1	−6	−13	42
1	1	−5	−18	24
2	1	−4	−21	0

$x = 2$ is one of the zeros of the function and the depressed polynomial is $x^2 - 4x - 21$.
Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} x^3 - 6x^2 - 13x + 42 &= (x^2 - 4x - 21)(x - 2) \\ &= (x - 7)(x + 3)(x - 2) \end{aligned}$$

The rational zeros are $x = -3, 2, 7$.

5. $f(x) = 2x^4 + 11x^3 + 26x^2 + 29x + 12$

SOLUTION:

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
$q: \pm 1, \pm 2$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Make a table for synthetic division and test possible zeros.

$\frac{p}{q}$	2	11	26	29	12
–1	2	9	17	12	0

$x = -1$ is one of the zeros of the function and the depressed polynomial is $2x^3 + 9x^2 + 17x + 12$.
Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} 2x^4 + 11x^3 + 26x^2 + 29x + 12 &= (2x^3 + 9x^2 + 17x + 12)(x + 1) \\ &= (2x + 3)(x^2 + 3x + 4)(x + 1) \end{aligned}$$

The polynomial $x^2 + 3x + 4$ does not have rational zeros. Therefore, the rational zeros are $x = -\frac{3}{2}, -1$.

Find all of the zeros of each function.

6. $f(x) = 3x^3 - 2x^2 - 8x + 5$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3},$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	3	−2	−8	5
$\frac{5}{3}$	3	3	−3	0

$x = \frac{5}{3}$ is one of the zeros of the function and the depressed polynomial is $3x^2 + 3x - 3$.

Factor the depressed polynomial and find the zeros.

$3x^2 + 3x - 3 = 0$ Write the depressed polynomial

$x^2 + x - 1 = 0$ Simplify.

$x = \frac{-1 \pm \sqrt{1+4}}{2}$ Use the Quadratic Formula to solve for x.

$= \frac{-1 \pm \sqrt{5}}{2}$ Simplify.

Therefore, the zeros of the polynomial are $x = \frac{5}{3}, \frac{-1 \pm \sqrt{5}}{2}$.

5-8 Rational Zero Theorem

7. $f(x) = 8x^3 + 14x^2 + 11x + 3$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	8	14	11	3
$-\frac{1}{2}$	8	10	6	0

$x = -\frac{1}{2}$ is one of the zeros of the function and the depressed polynomial is $8x^2 + 10x + 6$.

Factor the depressed polynomial and find the zeros.

$8x^2 + 10x + 6 = 0$	Write the depressed polynomial.
$x = \dfrac{-10 \pm \sqrt{100 - 192}}{16}$	Use the Quadratic formula.
$= \dfrac{-10 \pm \sqrt{-92}}{16}$	Simplify under the radical.
$= \dfrac{-10 \pm 2i\sqrt{23}}{16}$	Simplify radical.
$= \dfrac{-5 \pm i\sqrt{23}}{8}$	Simplify.

Therefore, the zeros of the polynomial are $x = -\frac{1}{2}, \frac{-5 \pm i\sqrt{23}}{8}$.

8. $f(x) = 4x^4 + 13x^3 - 8x^2 + 13x - 12$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	4	13	−8	13	−12
−4	4	−3	4	−3	0

$x = -4$ is one of the zeros of the function and the depressed polynomial is $4x^3 - 3x^2 + 4x - 3$.
Factor the depressed polynomial and find its zeros.

$$4x^3 - 3x^2 + 4x - 3 = 0$$
$$x^2(4x - 3) + 1(4x - 3) = 0$$
$$(x^2 + 1)(4x - 3) = 0$$
$$x^2 + 1 = 0 \quad \text{or} \quad 4x - 3 = 0$$
$$x^2 = -1 \qquad x = \frac{3}{4}$$
$$x = \pm i$$

The zeros of the polynomial are $x = -4, \frac{3}{4}, +i, -i.$

9. $f(x) = 4x^4 - 12x^3 + 25x^2 - 14x - 15$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{15}{4}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	4	−12	25	−14	−15
$-\frac{1}{2}$	4	−14	32	−30	0

$x = -\frac{1}{2}$ is one of the zeros of the function and the depressed polynomial is $4x^3 - 14x^2 + 32x - 30$.

Factor the depressed polynomial and find its zeros.

$$4x^3 - 14x^2 + 32x - 30 = 0$$
$$(2x-3)(2x^2 - 4x + 10) = 0$$

$$\begin{aligned} 2x^2 - 4x + 10 &= 0 \\ x^2 - 2x + 5 &= 0 \\ x &= \frac{2 \pm \sqrt{4-20}}{2} \\ x &= \frac{2 \pm 4i}{2} \\ &= 1 \pm 2i \end{aligned} \quad \text{or} \quad \begin{aligned} 2x - 3 &= 0 \\ x &= \frac{3}{2} \end{aligned}$$

The zeros of the polynomial are $x = -\frac{1}{2}, \frac{3}{2}, 1 \pm 2i$.

List all of the possible rational zeros of each function.

10. $f(x) = x^4 + 8x - 32$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –32 and q is a factor of 1.

$p: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$
$q: \pm 1$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$$

11. $f(x) = x^3 + x^2 - x - 56$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –56 and q is a factor of 1.

$p: \pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$
$q: \pm 1$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$$

12. $f(x) = 2x^3 + 5x^2 - 8x - 10$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –10 and q is a factor of 2.

$p: \pm 1, \pm 2, \pm 5, \pm 10$
$q: \pm 2$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}$$

13. $f(x) = 3x^6 - 4x^4 - x^2 - 35$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –35 and q is a factor of 3.

$$p: \pm1, \pm5, \pm7, \pm35$$
$$q: \pm1, \pm3$$

So, the possible rational zeros are:

$$\frac{p}{q} = \pm1, \pm5, \pm7, \pm35, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{7}{3}, \pm\frac{35}{3}$$

14. $f(x) = 6x^5 - x^4 + 2x^3 - 3x^2 + 2x - 18$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –18 and q is a factor of 6.

$$p: \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$$
$$q: \pm1, \pm2, \pm3, \pm6$$

So, the possible rational zeros are:

$$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$$

15. $f(x) = 8x^4 - 4x^3 - 4x^2 + x + 42$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 42 and q is a factor of 8.

$$p: \pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42$$
$$q: \pm1, \pm2, \pm4, \pm8$$

So, the possible rational zeros are:

$$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42, \pm\frac{1}{8}, \pm\frac{1}{4},$$
$$\pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{7}{8}, \pm\frac{7}{4}, \pm\frac{21}{8}, \pm\frac{21}{4}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$$

5-8 Rational Zero Theorem

16. $f(x) = 15x^3 + 6x^2 + x + 90$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 90 and q is a factor of 15.

$$p: \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 30, \pm 45, \pm 90$$
$$q: \pm 1, \pm 3, \pm 5, \pm 15$$

So, the possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90,$$
$$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{6}{5}, \pm \frac{9}{5}, \pm \frac{18}{5}, \pm \frac{1}{15}, \pm \frac{2}{15}$$

17. $f(x) = 16x^4 - 5x^2 + 128$

SOLUTION:

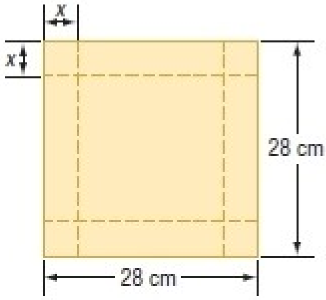
If $\frac{p}{q}$ is a rational zero, then p is a factor of 128 and q is a factor of 16.

$$p: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64 \pm 128$$
$$q: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$$

So, the possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64 \pm 128, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{1}{16}$$

18. **MANUFACTURING** A box is to be constructed by cutting out equal squares from the corners of a square piece of cardboard and turning up the sides.



a. Write a function $V(x)$ for the volume of the box.
b. For what value of x will the volume of the box equal 1152 cubic centimetres?
c. What will be the volume of the box if $x = 6$ centimetres?

SOLUTION:

a. The length and width of the box would be $(28 – 2x)$ and the height would be x.
The volume function of the box is:

$$\begin{aligned} V(x) &= (28-2x)(28-2x)x && \text{Volume} \\ &= (784-112x+4x^2)x && \text{Multiply.} \\ &= 784x-112x^2+4x^3 && \text{Multiply.} \\ &= 4x^3-112x^2+784x && \text{Reorder.} \end{aligned}$$

b. Substitute $V(x) = 1152$ cm^3 in the volume function.

$$\begin{aligned} 4x^3-112x^2+784x &= 1152 && \text{Volume of the box is 1152.} \\ x^3-28x^2+196x-288 &= 0 && \text{Subtract 1152 from each side and simplify.} \\ (x-2)(x-8)(x-18) &= 0 && \text{Factor} \\ x &= 2 \text{ or } 8 \text{ or } 18 && \text{Solve for } x. \end{aligned}$$

Since the length and width of the box are both equal to 28 cm, if $x = 18$, then $2x = 36$ which is greater than 28 cm. Therefore, the volume of the box will be equal to 1152 cubic centimeters when $x = 2$ or $x = 8$.

c. Substitute $x = 6$ in the volume function.

$$\begin{aligned} V(2) &= 4(6)^3 - 112(6)^2 + 784(6) \\ &= 864 - 4032 + 4704 \\ &= 1536 \text{ cm}^3 \end{aligned}$$

Find all of the rational zeros of each function.

19. $f(x) = x^3 + 10x^2 + 31x + 30$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	10	31	30
–1	1	9	22	8
–2	1	8	15	0

$x = –2$ is one of the zeros of the function and the depressed polynomial is $x^2 + 8x + 15$.

Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} x^3 + 10x^2 + 31x + 30 &= (x^2 + 8x + 15)(x + 2) \\ &= (x + 3)(x + 5)(x + 2) \end{aligned}$$

The rational zeros are $x = –5, –3, –2$.

20. $f(x) = x^3 - 2x^2 - 56x + 192$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 32, \pm 48, \pm 64, \pm 96, \pm 192$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	–2	–56	192
4	1	2	–48	0

$x = 4$ is one of the zeros of the function and the depressed polynomial is $x^2 + 2x - 48$.
Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} x^3 - 2x^2 - 56x + 192 &= (x^2 + 2x - 48)(x - 4) \\ &= (x - 6)(x + 8)(x - 4) \end{aligned}$$

The rational zeros are $x = 6, -8, 4$

21. $f(x) = 4x^3 - 3x^2 - 100x + 75$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm 25, \pm 75, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2},$$
$$\pm\frac{15}{2}, \pm\frac{25}{2}, \pm\frac{75}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{25}{4}, \pm\frac{75}{4}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	4	–3	–100	75
5	4	17	–15	0

$x = 5$ is one of the zeros of the function and the depressed polynomial is $4x^2 + 17x - 15$.
Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} 4x^3 - 3x^2 - 100x + 75 &= (4x^2 + 17x - 75)(x - 5) \\ &= (4x - 3)(x + 5)(x - 5) \end{aligned}$$

The rational zeros are $x = \frac{3}{4}, -5, 5.$

22. $f(x) = 4x^4 + 12x^3 - 5x^2 - 21x + 10$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	4	12	−5	−21	10
1	4	16	11	−10	0

$x = 1$ is one of the zeros of the function and the depressed polynomial is $4x^3 + 16x^2 + 11x - 10$.

$$4x^4 + 12x^3 - 5x^2 - 21x + 10 = (4x^3 + 16x^2 + 11x - 10)(x - 1)$$

Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} 4x^4 + 12x^3 - 5x^2 - 21x + 10 &= (4x^3 + 16x^2 + 11x - 10)(x - 1) \\ &= (x + 2)(2x + 5)(2x - 1)(x - 1) \end{aligned}$$

The rational zeros are $x = -2, -\frac{5}{2}, \frac{1}{2}, 1.$

23. $f(x) = x^4 + x^3 - 8x - 8$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	1	−8	0	−8
−1	1	0	−8	8	0

$x = -1$ is one of the zeros of the function and the depressed polynomial is $x^3 - 8x + 8$.

$$x^4 + x^3 - 8x - 8 = (x^3 - 8x + 8)(x + 1)$$

Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} x^4 + x^3 - 8x - 8 &= (x^3 - 8x + 8)(x + 1) \\ &= (x - 2)(x^2 + 2x - 4)(x + 1) \end{aligned}$$

The polynomial $x^2 + 2x - 4$ doesn't have rational zeros. Therefore, the rational zeros are $x = -1, 2.$

24. $f(x) = 2x^4 - 3x^3 - 24x^2 + 4x + 48$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	2	–3	–24	4	48
–2	2	–7	–10	24	0

$x = -2$ is one of the zeros of the function and the depressed polynomial is $2x^3 - 7x^2 - 10x + 24$.

$$2x^4 - 3x^3 - 24x^2 + 4x + 48 = (2x^3 - 7x^2 - 10x + 24)(x + 2)$$

Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$\begin{aligned} 2x^4 - 3x^3 - 24x^2 + 4x + 48 &= (2x^3 - 7x^2 - 10x + 24)(x + 2) \\ &= (x - 4)(2x - 3)(x + 2)(x + 2) \end{aligned}$$

The rational zeros are $x = 4, \frac{3}{2}, -2.$

25. $f(x) = 4x^3 + x^2 + 16x + 4$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{4}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	4	1	16	4
$-\frac{1}{4}$	4	0	16	0

$x = -\frac{1}{4}$ is one of the zeros of the function and the depressed polynomial is $4x^2 + 16$.

Again use the rational root theorem and synthetic division to find the zeros of the depressed polynomial.

$$4x^3 + x^2 + 16x + 4 = (4x^2 + 16)\left(x + \frac{1}{4}\right)$$

The polynomial $4x^2 + 16$ doesn't have rational zeros. Therefore, the rational zeros of the original polynomial are $x = -\frac{1}{4}$.

26. $f(x) = 81x^4 - 256$

SOLUTION:

$$\begin{aligned} 81x^4 - 256 &= \left(9x^2\right)^2 - (16)^2 \\ &= (9x^2 - 16)(9x^2 + 16) \\ &= (3x + 4)(3x - 4)(9x^2 + 16) \end{aligned}$$

The polynomial $9x^2 + 16$ doesn't have rational zeros.

Therefore, the rational zeros of the polynomial are: $x = -\frac{4}{3}, \frac{4}{3}$

Find all of the zeros of each function.

27. $f(x) = x^3 + 3x^2 - 25x + 21$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm 7, \pm 21$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	3	−25	21
1	1	4	−21	0

$x = 1$ is one of the zeros of the function and the depressed polynomial is $x^2 + 4x - 21$.
Factor the depressed polynomial and find the zeros.

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ (x-3)(x+7) &= 0 \\ x &= 3, -7 \end{aligned}$$

Therefore, the zeros of the polynomial are x = 1, 3, –7.

28. $f(x) = 6x^3 + 5x^2 - 9x + 2$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	6	5	–9	2
$\frac{2}{3}$	6	9	–3	0

$x = \frac{2}{3}$ is one of the zeros of the function and the depressed polynomial is $6x^2 + 9x - 3$.

Find the zeros of the depressed polynomial using the quadratic formula.

$6x^2 + 9x - 3 = 0$	Depressed polynomial
$x = \frac{-9 \pm \sqrt{81+72}}{12}$	Use the Quadratic Formula
$= \frac{-9 \pm 3\sqrt{17}}{12}$	Simplify under the radical.
$= \frac{-3 \pm \sqrt{17}}{4}$	Simplify.

Therefore, the zeros of the polynomial are $x = \frac{2}{3}, \frac{-3 \pm \sqrt{17}}{4}$.

29. $f(x) = x^4 - x^3 - x^2 - x - 2$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	−1	−1	−1	−2
−1	1	−2	1	−2	0

$x = -1$ is one of the zeros of the function and the depressed polynomial is $x^3 - 2x^2 + x - 2$.
Factor the depressed polynomial and find its zeros.

$$x^3 - 2x^2 + x - 2 = 0$$
$$x^2(x-2) + 1(x-2) = 0$$
$$(x^2+1)(x-2) = 0$$
$$x^2 + 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x^2 = -1 \qquad\qquad x = 2$$
$$x = \pm i$$

The zeros of the polynomial are $x = -1, 2, +i, -i$.

5-8 Rational Zero Theorem

30. $f(x) = 10x^3 - 17x^2 - 7x + 2$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{1}{10}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	10	−17	−7	2
2	10	3	−1	0

$x = 2$ is one of the zeros of the function and the depressed polynomial is $10x^2 + 3x - 1$.
Factor the depressed polynomial and find its zeros.

$10x^2 + 3x - 1 = 0$	Factor the depressed polynomial.
$10x^2 + 5x - 2x - 1 = 0$	Rewrite $3x$.
$5x(2x+1) - (2x+1) = 0$	Distributive Property
$(5x-1)(2x+1) = 0$	Factor by grouping.
$x = \frac{1}{5}; -\frac{1}{2}$	Solve for x.

The zeros of the polynomial are $x = 2, \frac{1}{5}; -\frac{1}{2}$.

31. $f(x) = x^4 - 3x^3 + x^2 - 3x$

SOLUTION:

The possible rational zeros are:

$\frac{p}{q}: \pm 1, \pm 3$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	−3	+1	−3	0
3	1	0	1	0	0

$x = 3$ is one of the zeros of the function and the depressed polynomial is $x^3 + x$.
Factor the depressed polynomial and find its zeros.

$x^3 + x = 0$	Depressed polynomial
$x(x^2 + 1) = 0$	Factor.
$x = 0$ or $x^2 + 1 = 0$	Set each factor to zero.
$x = 0$ or $x = \pm i$	Solve

The zeros of the polynomial are $x = 3, 0, +i, -i$.

32. $f\ (x) = 6x^3 + 11x^2 - 3x - 2$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	6	11	–3	–2
–2	6	–1	–1	0

$x = -2$ is one of the zeros of the function and the depressed polynomial is $6x^2 - x - 1$.
Factor the depressed polynomial and find its zeros.

$$6x^2 - x - 1 = 0 \quad \text{Factor depressed polyonomial}$$

$$6x^2 - 3x + 2x - 1 = 0 \quad \text{Rewrite the middle term.}$$

$$3x(2x-1) + (2x-1) = 0 \quad \text{Distributive Property}$$

$$(2x-1)(3x+1) = 0 \quad \text{Factor by grouping}$$

$$x = \frac{1}{2}, -\frac{1}{3} \quad \text{Solve each factor}$$

The zeros of the polynomial are $x = -2, \frac{1}{2}, -\frac{1}{3}$.

33. $f(x) = 6x^4 + 22x^3 + 11x^2 - 38x - 40$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1,\ \pm 2,\ \pm 4,\ \pm 5,\ \pm 8,\ \pm 10,\ \pm 20,\ \pm 40,\ \pm\frac{1}{2},\ \pm\frac{5}{2},$$
$$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{5}{3}, \pm\frac{8}{3}, \pm\frac{10}{3}, \pm\frac{20}{3}, \pm\frac{40}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	6	22	11	−38	−40
−2	6	10	−9	−20	0

$x = -2$ is one of the zeros of the function and the depressed polynomial is $6x^3 + 10x^2 - 9x - 20$. Factor the depressed polynomial and find its zeros.

$$6x^3 + 10x^2 - 9x - 20 = 0$$
$$(3x - 4)(2x^2 + 6x + 5) = 0$$
$$3x - 4 = 0 \quad \text{or} \quad 2x^2 + 6x + 5 = 0$$
$$x = \frac{4}{3} \qquad x = \frac{-6 \pm \sqrt{36 - 40}}{4}$$
$$= \frac{-3 \pm i}{2}$$

The zeros of the polynomial are $x = -2, \frac{4}{3}, \frac{-3 \pm i}{2}$.

34. $f(x) = 2x^3 - 7x^2 - 8x + 28$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28, \pm\frac{1}{2}, \pm\frac{7}{2}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	2	−7	−8	+28
2	2	−3	−14	0

$x = 2$ is one of the zeros of the function and the depressed polynomial is $2x^2 - 3x - 14$.
Factor the depressed polynomial and find its zeros.

$$\begin{aligned} 2x^2 - 3x - 14 &= 0 && \text{Factor the depressed polynomial} \\ 2x^2 + 4x - 7x - 14 &= 0 && \text{Rewrite the middle term.} \\ 2x(x+2) - 7(x+2) &= 0 && \text{Distributive Property} \\ (2x-7)(x+2) &= 0 && \text{Factor by grouping} \\ x &= \frac{7}{2}, -2 && \text{Solve each factor} \end{aligned}$$

The zeros of the polynomial are $x = 2, -2, \frac{7}{2}$.

5-8 Rational Zero Theorem

35. $f(x) = 9x^5 - 94x^3 + 27x^2 + 40x - 12$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	9	0	−94	27	40	−12
3	9	27	−13	−12	4	0

$x = 3$ is one of the zeros of the function and the depressed polynomial is $9x^4 + 27x^3 - 13x^2 - 12x + 4$.
Factor the depressed polynomial and find its zeros.

$$9x^4 + 27x^3 - 13x^2 - 12x + 4 = 0$$

Factor the Depressed Polynomial.

$$(3x+2)(3x-2)(x^2+3x-1) = 0$$

$$3x + 2 = 0 \quad x = -\frac{2}{3} \quad \text{or} \quad 3x - 2 = 0 \quad x = \frac{2}{3} \quad \text{or} \quad x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$$

The zeros of the polynomial are $x = 3, \frac{2}{3}, -\frac{2}{3}, \frac{-3 \pm \sqrt{13}}{2}$.

36. $f(x) = x^5 - 2x^4 - 12x^3 - 12x^2 - 13x - 10$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	−2	−12	−12	−13	−10
−1	1	−3	−9	−3	−10	0

$x = -1$ is one of the zeros of the function and the depressed polynomial is $x^4 - 3x^3 - 9x^2 - 3x - 10$.
Factor the depressed polynomial and find its zeros.

$$x^4 - 3x^3 - 9x^2 - 3x - 10 = 0 \quad \text{Factor the Depressed Polynomial.}$$

$$(x^2 + 1)(x^2 - 3x - 10) = 0 \quad \text{Rewrite middle term.}$$

$$x^2 + 1 = 0 \quad \text{or} \quad x^2 - 3x - 10 = 0$$

$$x = \pm i \qquad (x - 5)(x + 2) = 0$$

$$x = 5, \ -2 \quad \text{Solve each factor.}$$

The zeros of the polynomial are $x = -1, -2, 5, i, -i$.

37. $f(x) = 48x^4 - 52x^3 + 13x - 3$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{12}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{24}, \pm\frac{1}{48}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	48	−52	0	13	−3
$\frac{1}{2}$	48	−28	−14	6	0

$x = \frac{1}{2}$ is one of the zeros of the function and the depressed polynomial is $48x^3 - 28x^2 - 14x + 6$.

Factor the depressed polynomial and find its zeros.

$$48x^3 - 28x^2 - 14x + 6 = 0 \qquad \text{Factor the Depressed Polynomial.}$$
$$24x^3 - 14x^2 - 7x + 3 = 0 \qquad \text{Factor by grouping.}$$
$$(2x+1)(3x-1)(4x-3) = 0 \qquad \text{Solve each factor}$$
$$x = -\frac{1}{2}, \frac{1}{3}, \frac{3}{4}$$

The zeros of the polynomial are $x = -\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{4}$.

38. $f(x) = 5x^4 - 29x^3 + 55x^2 - 28x$

SOLUTION:

First factor x from the original polynomial.

$$\begin{aligned} f(x) &= x(5x^3 - 29x^2 + 55x - 28) \\ &= x(5x-4)(x^2 - 5x + 7) \end{aligned}$$

Find the zeros of the polynomial.

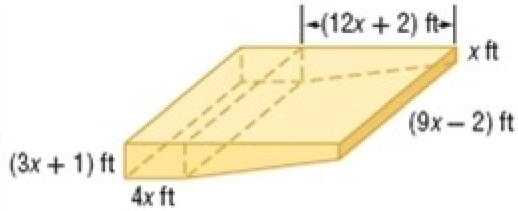
$$x = 0 \quad \text{or} \quad 5x - 4 = 0 \quad \text{or} \quad x^2 - 5x + 7 = 0$$

$$x = \frac{4}{5} \qquad x = \frac{5 \pm \sqrt{25-28}}{2}$$

$$x = \frac{5 \pm i\sqrt{3}}{2}$$

The zeros of the polynomial are $x = 0, \frac{4}{5}, \frac{5 \pm i\sqrt{3}}{2}$.

39. **SWIMMING POOLS** A diagram of the swimming pool at the Midtown Community Center is shown below. The pool can hold 9175 cubic feet of water.



a. Write a polynomial function that represents the volume of the swimming pool.

b. What are the possible values of x? Which of these values are reasonable?

SOLUTION:

a. $V(x) = 324x^3 + 54x^2 - 19x - 2$

b. $1.05i$, $-4.22i$, 3; 3 is the only reasonable value for x. The other two values are imaginary.

40. **CCSS MODELING** A portion of the path of a certain roller coaster can be modeled by $f(t) = t^4 - 31t^3 + 308t^2 - 1100t + 1200$ where t represents the time in seconds and $f(t)$ represents when the height of the roller coaster is at a relative maximum. Use the Rational Zero Theorem to determine the four times at which the roller coaster is at ground level.

SOLUTION:

From the Rational Zero Theorem, the possible zeros will be factors of 1200.

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 10, \pm 12, \pm 16, \pm 20, \pm 24, \pm 30, \pm 40,$
$\pm 50, \pm 60, \pm 75, \pm 120, \pm 150, \pm 200, \pm 300, \pm 400, \pm 600, \pm 1200$

There are at most 4 roots since the degree of the polynomial is 4.
$t = 1$ is not a zero, however, 2 is.
2 s, 4 s, 10 s, and 15 s

41. **FOOD** A restaurant orders spaghetti sauce in cylindrical metal cans. The volume of each can is about 160π cubic inches, and the height of the can is 6 inches more than the radius.

a. Write a polynomial equation that represents the volume of a can. Use the formula for the volume of a cylinder, $v = \pi r^2 h$

b. What are the possible values of *r*? Which of these values are reasonable for this situation?

c. Find the dimensions of the can.

SOLUTION:

a. Let *r* be the radius of the cylindrical can. The height of the can would be 6 + *r*.
Therefore, the volume of the can *V*(*r*) is:

$$\begin{aligned} V(r) &= \pi r^2(6+r) \\ &= 6\pi r^2 + \pi r^3 \end{aligned}$$

b. Set $V(r) = 160\pi$ in the volume function. To find the possible values of *r*, find the possible zeros of the polynomial equation.

$160\pi = 6\pi r^2 + \pi r^3$	Volume function
$r^3 + 6r^2 - 160 = 0$	Divide each term by π.
$(r-4)(x^2+10x+40) = 0$	Factor by grouping.

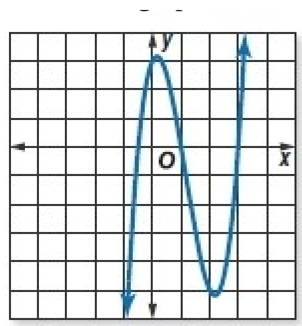
$$r - 4 = 0 \quad \text{or} \quad x^2 + 10x + 40 = 0$$
$$r = 4$$

$x = \dfrac{-10 \pm \sqrt{100-160}}{2}$	Solve using the Quadratic Formula.
$= \dfrac{-10 \pm \sqrt{-60}}{2}$	Simplify under the radical.
$= \dfrac{-10 \pm 2i\sqrt{15}}{2}$	Simplify the radical.
$= -5 \pm i\sqrt{15}$	Simplify.

The possible values of *r* are $4, -5 \pm i\sqrt{15}$. Since the radius cannot be a complex number, the value of *r* must be 4.

c. Radius of the can *r* = 4 in.
Height of the can *h* = 6 + 4 = 10 in.

42. Refer to the graph.



a. Find all of the zeros of $f(x) = 2x^3 + 7x^2 + 2x - 3$ and $g(x) = 2x^3 - 7x^2 + 2x + 3$.
b. Determine which function, f or g, is shown in the graph at the right.

SOLUTION:

a. Zeros of $f(x)$:
The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	2	7	2	−3
−1	2	5	−3	0

$x = -1$ is one of the zeros of the function and the depressed polynomial is $2x^2 + 5x - 3$.
Factor the depressed polynomial and find its zeros.

$$\begin{aligned} 2x^2 + 5x - 3 &= 0 \\ x &= \frac{-5 \pm \sqrt{25 + 24}}{4} \\ &= \frac{-5 \pm 7}{4} \\ &= \frac{1}{2}, -3 \end{aligned}$$

The zeros of the polynomial $f(x)$ are $x = -1, -3, \frac{1}{2}$.

Zeros of $g(x)$:
The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	2	−7	2	3
1	2	−5	−3	0

$x = 1$ is one of the zeros of the function and the depressed polynomial is $2x^2 - 5x - 3$. Factor the depressed polynomial and find its zeros.

$$2x^2 - 5x - 3 = 0$$
$$\begin{aligned} x &= \frac{5 \pm \sqrt{25 + 24}}{4} \\ &= \frac{5 \pm 7}{4} \\ &= -\frac{1}{2}, 3 \end{aligned}$$

The zeros of the polynomial $g(x)$ are $x = 1, 3, -\frac{1}{2}$.

b. From the graph, the zeros of function are at $x = 1, 3, -\frac{1}{2}$. Therefore, it is the graph of $g(x)$.

43. **MUSIC SALES** Refer to the beginning of the lesson.

a. Write a polynomial equation that could be used to determine the year in which music sales would be about \$20,000,000,000.

b. List the possible whole number solutions for your equation in part a.

c. Determine the approximate year in which music sales will reach \$20,000,000,000.

d. Does the model represent a realistic estimate for all future music sales? Explain your reasoning.

SOLUTION:

a. The music sales are about \$9000 millions in x years.
Therefore, the equation of the polynomial could be

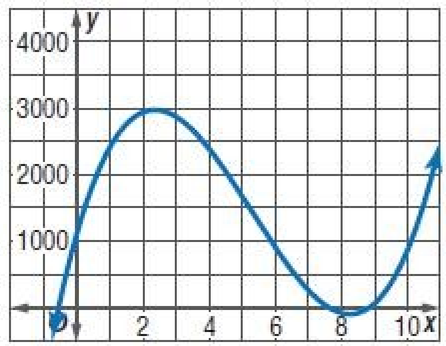
$$9000 = 30x^3 - 478x^2 + 1758x + 10,092$$

$30x^3 - 478x^2 + 1758x + 10,092 - 9000 = 0$

$30x^3 - 478x^2 + 1758x + 1092 = 0$

b. The possible whole number zeros are: 1, 2, 3, 4, 6, 7, 12, 13, 14, 21, 26, 28, 39, 42, 52, 78, 84, 91, 156, 182, 273, 364, 546, 1092.

c. Graph the function $f(x) = 30x^3 - 478x^2 + 1758x + 1092$.



The graph intersects the x-axis near 8. Therefore, the music sales each \$9000 billion in about 8 years.
The year in which music sales reach \$9000 billion is 2005 + 8 or 2013.

d. No; Sample answer: Music sales fluctuate from 2005 to 2015, then increase indefinitely. It is not reasonable to expect sales to increase forever.

Find all of the zeros of each function.

44. $f(x) = x^5 + 3x^4 - 19x^3 - 43x^2 + 18x + 40$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	3	−19	−43	18	40
−2	1	1	−21	−1	20	0

$x = -2$ is one of the zeros of the function and the depressed polynomial is $x^4 + x^3 - 21x^2 - x + 20$. Factor the depressed polynomial and find its zeros.

$$x^4 + x^3 - 21x^2 - x + 20 = 0$$
$$(x^2 - 1)(x^2 + x - 20) = 0$$
$$x^2 - 1 = 0 \quad \text{or} \quad x^2 + x - 20 = 0$$
$$x = \pm 1 \qquad (x - 4)(x + 5) = 0$$
$$x = 4, -5$$

The zeros of the polynomial are $x = -2, -1, 1, 4, -5$.

45. $f(x) = x^5 - x^4 - 23x^3 + 33x^2 + 126x - 216$

SOLUTION:

The possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18,$$
$$\pm 24, \pm 27, \pm 36, \pm 54, \pm 72, \pm 108, \pm 216$$

Test for some possible zeros using synthetic division.

$\frac{p}{q}$	1	–1	–23	33	126	–216
2	1	1	–21	–9	108	0

$x = 2$ is one of the zeros of the function and the depressed polynomial is $x^4 + x^3 - 21x^2 - 9x + 108$.
Factor the depressed polynomial and find its zeros.

$$x^4 + x^3 - 21x^2 - 9x + 108 = 0$$
$$(x^2 - 9)(x^2 + x - 12) = 0$$
$$x^2 - 9 = 0 \quad \text{or} \quad x^2 + x - 12 = 0$$
$$x = \pm 3 \qquad (x - 3)(x + 4) = 0$$
$$x = 3, -4$$

The zeros of the polynomial are $x = 2, 3, 3, -3, -4$.

46. **CCSS CRITIQUE** Doug and Mika are listing all of the possible rational zeros for $f(x) = 4x^4 + 8x^5 + 10x^2 + 3x + 16$. Is either of them correct? Explain your reasoning.

Doug

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}$

Mika

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$

SOLUTION:

Sample answer: Doug; the value of q is the leading coefficient, which is 4, not 8.

47. **CHALLENGE** Give a polynomial function that has zeros at and $5 + 2i$.

SOLUTION:

Sample answer: $f(x) = x^4 - 12x^3 + 47x^2 - 38x - 58$

48. **REASONING** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning. *If all of the possible zeros of a polynomial function are integers, then the leading coefficient of the function is* 1 or -1.

SOLUTION:

Sample answer: Always; in order for the possible zeros of a polynomial function to be integers, the value of q must be 1 or -1. Otherwise, the possible zeros could be a fraction. In order for q to be 1 or -1, the leading coefficient of the polynomial must also be 1 or -1

49. **OPEN ENDED** Write a function that has possible zeros of $\pm 18, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1, \pm \frac{9}{4},$
$\pm \frac{9}{2}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{1}{2}$ *and* $\pm \frac{1}{4}$.

SOLUTION:

Sample answer: $f(x) = 4x^5 + 3x^3 + 8x + 18$

50. **CHALLENGE** The roots of $x^2 + bx + c = 0$ are M and N. If $|M - N| = 1$, express c in terms of b.

SOLUTION:

The roots of the equation are $\frac{-b \pm \sqrt{b^2 - 4c}}{2}$.

Now,

$$|M - N| = 1$$
$$\left|\frac{-b + \sqrt{b^2 - 4c}}{2} - \frac{-b - \sqrt{b^2 - 4c}}{2}\right| = 1$$
$$\left|\frac{2\sqrt{b^2 - 4c}}{2}\right| = 1$$
$$b^2 - 4c = 1$$
$$c = \frac{b^2 - 1}{4}$$

51. **WRITING IN MATH** Explain the process of using the Rational Zero Theorem to determine the number of possible rational zeros of a function.

SOLUTION:

Sample answer: For any polynomial function, the constant term represents p and the leading coefficient represents q. The possible zeros of the function can be found with $\pm\frac{p}{q}$ where the fraction is every combination of factors of p and q. For example, if p is 4 and q is 3, then $\pm 4, \pm 2, \pm 1, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$, and $\pm\frac{1}{3}$ are all possible zeros.

52. **ALGEBRA** Which of the following is a zero of the function $f(x) = 12x^5 - 5x^3 + 2x - 9$?

A –6

B $-\frac{2}{3}$

C $\frac{3}{8}$

D 1

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of –9 and q is a factor of 12.

$p: \pm 1, \pm 3, \pm 9$
$q: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

So, the possible rational zeros are:

$$\frac{p}{q}: \pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$$

Among the choices, the choice D is correct.

53. **SAT/ACT** How many negative real zeros does $f(x) = x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$ have?

F 5

G 3

H 2

J 1

K 0

SOLUTION:

Use Descartes's Rule.

$f(-x) = -x^5 - 2x^4 + 4x^3 + 4x^2 + 5x + 6$

There is only one sign change. So, there is exactly one negative zero.
The correct choice is J.

54. **ALGEBRA** For all nonnegative numbers n,

let $\boxed{n} = \frac{\sqrt{n}}{2}$. if $\boxed{n}$ = 4, what is the value of n?

A 2
B 4
C 16
D 64

SOLUTION:

Substitute $\boxed{n} = 4$ in the expression.

$$4 = \frac{\sqrt{n}}{2}$$
$$\sqrt{n} = 8$$
$$n = 8^2$$
$$= 64$$

The correct choice is D.

55. **GRIDDED RESPONSE** What is the y-intercept of a line that contains the point (–1, 4) and has the same x-intercept as $x + 2y = -3$?

SOLUTION:

The x-intercept of the line is –3. Therefore, the line must pass through the point (–3, 0).
So, the equation of the line passes through the points (–1, 4) and (–3, 0) is:

$$\frac{y-4}{0-4} = \frac{x+1}{-3+1}$$
$$y = 2x + 6$$

The y-intercept of the line is 6.

Write a polynomial function of least degree with integral coefficients that has the given zeros.

56. 6, –3, $\sqrt{2}$

SOLUTION:

The irrational roots occur in conjugate pairs. Therefore, $-\sqrt{2}$ is also a root of the polynomial.
So,

$$(x-6)(x+3)(x-\sqrt{2})(x+\sqrt{2}) = (x^2 - 3x - 18)(x^2 - 2)$$
$$= x^4 - 2x^2 - 3x^3 + 6x - 18x^2 + 36$$
$$= x^4 - 3x^3 - 20x^2 + 6x + 36$$

57. 5, –1, 4*i*

SOLUTION:

The complex roots occur in conjugate pairs.
Therefore,

$$\begin{aligned}(x-5)(x+1)(x-4i)(x+4i) &= (x^2-4x-5)(x^2+16)\\ &= x^4+16x^2-4x^3-64x-5x^2-80\\ &= x^4-4x^3+11x^2-64x-80\end{aligned}$$

58. –4, –2, $i\sqrt{2}$

SOLUTION:

The complex roots occur in conjugate pairs.
Therefore,

$$\begin{aligned}(x+4)(x+2)(x-i\sqrt{2})(x+i\sqrt{2}) &= (x^2+6x+8)(x^2+2)\\ &= x^4+2x^2+6x^3+12x+8x^2+16\\ &= x^4+6x^3+10x^2+12x+16\end{aligned}$$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

59. $x^4 + 5x^3 + 5x^2 - 5x - 6$; $x + 3$

SOLUTION:

$x + 3$ is a factor.

So, $x^4+5x^3+5x^2-5x-6=(x^3+2x^2-x-2)(x+3)$.

The possible rational zeros of $P(x)=x^3+2x^2-x-2$ are ±1, and ±2.
Test the possible zeros.

$$\begin{aligned}P(1) &= 1^3+2(1)^2-1-2\\ &= 0\\ P(-1) &= (-1)^3+2(-1)^2+1-2\\ &= 0\\ P(2) &= (2)^3+2(2)^2-2-2\\ &\neq 0\\ P(-2) &= (-2)^3+2(-2)^2+2-2\\ &= 0\end{aligned}$$

Therefore, $x = 1$, $x = -1$ and $x = -2$ are the zeros of the function.

$$x^4+5x^3+5x^2-5x-6=(x-1)(x+1)(x+2)(x+3)$$

The remaining factors are: $(x - 1)(x + 2)(x + 1)$

60. $a^4 - 2a^3 - 17a^2 + 18a + 72$; $a - 3$

SOLUTION:

$a^4 - 2a^3 - 17a^2 + 18a + 72 = (a^3 + a^2 - 14a - 24)(a - 3)$

The possible rational zeros of $P(a) = a^3 + a^2 - 14a - 24$ is $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$
Test the possible zeros:

$$\begin{aligned} P(-2) &= (-2)^3 + (-2)^2 - 14(-2) - 24 \\ &= 0 \\ P(-3) &= (-3)^3 + (-3)^2 - 14(-3) - 24 \\ &= 0 \\ P(4) &= (4)^3 + (4)^2 - 14(4) - 24 \\ &= 0 \end{aligned}$$

Therefore, $a = -2$, $a = -3$ and $a = 4$ are the zeros of the function.

$a^4 - 2a^3 - 17a^2 + 18a + 72 = (a + 2)(a + 3)(a - 4)(a - 3)$

The remaining factors are: $(a + 2)(a + 3)(a - 4)$

61. $x^4 + x^3 - 11x^2 + x - 12$; $x + i$

SOLUTION:

Since the complex roots occur in conjugate pairs, $(x - i)$ is also a root of the polynomial.

$$\begin{aligned} x^4 + x^3 - 11x^2 + x - 12 &= (x^2 + x - 12)(x + i)(x - i) \\ &= (x - 3)(x + 4)(x + i)(x - i) \end{aligned}$$

The remaining factors are: $(x - 3)(x + 4)(x - i)$

62. **BRIDGES** The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function y = $0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$.
a. Calculate the value of the discriminate.
b. What does the discriminate tell you about the supporting cables of the Golden Gate Bridge?

SOLUTION:

a. The discriminate of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.
Here, a = 0.00012, b = 0, and $c = 6$.
The value of the discriminate is:
$(0)^2 - 4(0.00012)(6) = -0.00288$

b. Sample answer: This means that the cables do not touch the floor of the bridge, since the graph does not intersect the x-axis and the roots are imaginary.

63. **RIDES** An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground.

a. Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet, to find how long the riders are in free-fall.

b. Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

SOLUTION:

a. Here, $h(t) = 30$ ft and $h_0 = 225$ ft.

$$\begin{aligned} 30 &= -16t^2 + 225 \\ 16t^2 &= 195 \\ t^2 &= \frac{195}{16} \\ &= 12.1875 \\ t &\approx 3.5 \text{ seconds} \end{aligned}$$

b. Substitute $t = 5$ seconds and $h(\mathrm{t}) = 30$.

$$\begin{aligned} 30 &= -16(5)^2 + h_0 \\ 30 &= -400 + h_0 \\ h_0 &= 430 \text{ ft} \end{aligned}$$

The height of the tower should be 430 ft.

Simplify.

64. $(x - 4)(x + 3)$

SOLUTION:

$$\begin{aligned} (x-4)(x+3) &= x^2 + 3x - 4x - 12 \\ &= x^2 - x - 12 \end{aligned}$$

65. $3x(x^2 + 4)$

SOLUTION:

$$\begin{aligned} 3x(x^2+4) &= 3x\left(x^2\right) + 3x\left(4\right) \\ &= 3x^3 + 12x \end{aligned}$$

66. $x^2(x - 2)(x + 1)$

SOLUTION:

$$\begin{aligned} x^2(x-2)(x+1) &= x^2(x^2 - x - 2) \\ &= x^4 - x^3 - 2x^2 \end{aligned}$$

Find each value if $f(x) = 6x + 2$ and $g(x) = -4x^2$.

67. $f(5)$

SOLUTION:

$$\begin{aligned} f(5) &= 6(5) + 2 \\ &= 30 + 2 \\ &= 32 \end{aligned}$$

68. $g(-3)$

SOLUTION:

$$\begin{aligned} g(-3) &= -4(-3)^2 \\ &= -4(9) \\ &= -36 \end{aligned}$$

69. $f(3c)$

SOLUTION:

$$\begin{aligned} f(3c) &= 6(3c) + 2 \\ &= 18c + 2 \end{aligned}$$